Heavy pentaquarks

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We construct the spin-flavor wave functions of the possible heavy pentaquarks containing an anticharm or antibottom quark using various clustered quark models. Then we estimate the masses and magnetic moments of the $J^P=\frac{1}{2}^+$ or $\frac{3}{2}^+$ heavy pentaquarks. We emphasize the difference in the predictions of these models. Future experimental searches at BESIII, CLEOc, BELLE, and LEP may find these interesting states.

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I. INTRODUCTION

After LEPS Collaboration [1] announced the discovery of the narrow Θ^+ pentaquark state around 1540 MeV, many other experimental groups [2–8] claimed that they confirmed the existence of this exotic baryon with the minimal quark content $uudd\bar{s}$. The third component of its isospin is $I_z=0$. The pK^+ spectrum is featureless [3,4,6,7]. So the Θ^+ pentaquark is an iso-scalar if it is really a member of the anti-decuplet. At present, the possibility of this state being a member of another multiplet is not completely excluded. Hence its total isospin is probably zero. All the other quantum numbers remain undetermined. Later, another narrow pentaquark candidate Ξ_5^{--} with strangeness $S=-2$, baryon number $B=1$, and isospin $I=\frac{3}{2}$ around 1862 MeV was observed by NA49 Collaboration [9]. However this state has not been confirmed by other groups, and its existence is not established up to now [10].

These exotic baryons are definitely beyond the conventional quark model, in which ordinary baryons are composed of three quarks and mesons are composed of a pair of quark and antiquark. Although the simple quark model has been extremely successful in the classification of hadron states, its foundation has not been derived from quantum chromodynamics (QCD) so far. QCD allows a much richer spectrum than that in quark model. For example, nonconventional hadrons such as glueballs, hybrid mesons, other multiquark hadrons are expected in QCD. Convinced that the quark model cannot be the whole story, people have been looking for these exotic hadrons for decades. None of them was established without controversy [11] until the discovery of the Θ^+ pentaquark.

Theoretical study of pentaquark states dated back to the early days of QCD using MIT bag model [12]. A few years ago, Diakonov *et al.* predicted the masses and widths of the antidecuplet baryons using the chiral soliton model (CSM) and suggested several reaction channels to look for them [13], which partly motivated the experimental search. But the resulting masses and widths of the antidecuplet baryons are very sensitive to the inputs in this model [13–15]. For example, either adopting the commonly used value 45 MeV for the σ-term or identifying $N(1710)$ as a member of the antidecuplet will lead to a Ξ_5^{--} pentaquark with a mass of 2070 MeV, which is 210 MeV higher than that observed by NA49 Collaboration [13,16]. Very recently, Ellis *et al.* used the new value (79 ± 7) MeV and (64 ± 7) MeV from two recent analysis [17] for the σ-term and obtained a fairly good description of both Θ^+ and Ξ^{--} masses [15]. But the theoretical foundation of the treatment of the pentaquarks in the chiral soliton model is challenged with the large N_c formalism by Refs. [18,19].

Since early last year, there appeared many theoretical papers trying to interpret these exotic states. Among them, Jaffe and Wilczek's (JW) diquark model is a typical one [20]. In their model, the Θ^+ pentaquark is composed of a pair of diquarks and a strange antiquark. The flavor antidecuplet is always accompanied by an octet which is nearly degenerate and will mix with the decuplet. Shuryak and Zahed's suggested that the pentaquark mass be lower by replacing one scalar diquark with one tensor diaquark in JW's model [21]. Karliner and Lipkin's (KL) proposed a diquark-triquark model which also led to a flavor antidecuplet and octet [22]. In all the above clustered quark model, the resulting angular momentum and parity J^P of the pentaquark can be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. Dudek and Close first estimated the $J=\frac{3}{2}\Theta^+$ pentaquark mass in JW's and KL's model by considering the spin-orbital force [23].

Many models (e.g., [24]) including all the above clustered quark models were constructed to ensure the pentaquarks possess positive parity as in the original chiral soliton model [13,14]. But the Θ^+ pentaquark parity is still a pending issue. For example, QCD sum rule approach [25,26] and lattice QCD simulation favor negative parity [27]. Some other models favor negative parity as well [28,29]. Recently, many theoretical papers proposed interesting ways to determine Θ^+ parity [30–37].

In a recent paper [38], Jaffe and Wilczek pointed out that the decay mode $\Xi_5\to\Xi^*+\pi$ observed by NA49 [9] signals the existence of an octet around 1862 MeV together with the anti-decuplet since the latter cannot decay into a decuplet and an octet in the $SU(3)_f$ symmetry limit. If further confirmed, this experiment disfavors a $J^P=\frac{1}{2}^-$ assignment for

Ξ_5^{--} and poses a serious challenge for the chiral soliton model since there is no baryon pentaquark octet in the rotational band in this model. But it may be hard to exclude it since there always exist excited vibrational octet modes. These modes cannot be calculated rigourously within the chiral soliton model [15].

Besides its mass, the pentaquark magnetic moment is another very important quantity encoding the underlying quark structure and dynamics. Practically speaking, it is essential in the calculation of the cross-sections of the pentaquark electro-and photoproduction processes. Different models may yield the same mass, especially when experimental data are available. But they may yield very different results for the magnetic moment, which provides a very good way to distinguish various models.

We have employed the light cone QCD sum rule technique to calculate the Θ^+ pentaquark magnetic moment [39]. In Refs. [40,41] we have calculated magnetic moments of both $\frac{1}{2}^+$ and $\frac{3}{2}^+$ octet and antidecuplet pentaquark states in the framework of various clustered quark models. Several other groups have also discussed the light pentaquark magnetic moments, especially μ_{Θ^+} [42–45].

In this work we extend the same formalism to discuss the heavy pentaquarks containing an anticharm or antibottom quark systematically. In the framework of the above clustered quark models, we get one $SU(3)_f$ antisextet in JW's model, both an anti-sextet and triplet in KL's models with either $J^P=\frac{1}{2}^+$ or $\frac{3}{2}^+$. Then we estimate their masses and magnetic moments. Experimentally, several groups have proposed to hunt heavy pentaquarks at LEP and B factories [46], which calls for theoretical efforts on this topic. There have been some discussions of heavy pentaquarks in Refs. [20,22,23,47].

As in the case of conventional heavy hadrons, the presence of the heavy antiquark makes the treatment of the system simpler. If the light quarks are really strongly correlated as proposed in the clustered quark models, the heavy pentaquark system is the ideal place to study this kind of correlation without the additional complication due to the extra light antiquark. Exploration of these heavy exotic states will deepen our knowledge of strong interactions.

This paper is organized as follows: In Sec. I we review this rapidly developing field briefly. In the following sections, we calculate the masses and magnetic moments using various clustered quark models. Finally, we compare the results from different models and discuss the experimental searches of these interesting heavy pentaquarks.

II. HEAVY PENTAQUARK STATES IN JAFFE AND WILCZEK'S MODEL

Jaffe and Wilczek proposed that there exists strong correlation between the light quark pair when they are in the antisymmetric color ($\bar{3}_c$), flavor ($\bar{3}_f$), isospin ($I=0$) and spin ($J=0$) configuration [20,38]. The lighter the quarks, the stronger the correlation, which helps the light quark pair form a diquark. For example, the ud diquark behaves like a scalar with positive parity. Such correlation may arise from the color spin force from the one gluon exchange or the flavor spin force induced by the instanton interaction. In order to accommodate the Θ^+ pentaquark, Jaffe and Wilczek required the flavor wave function of the diquark pair to be symmetric $\bar{\mathbf{6}}_\mathbf{f}$ and their color wave function to be antisymmetric $\mathbf{3}_c$. Bose statistics of the scalar diquarks demands an odd orbital excitation between the two diquarks while there is no orbital excitation inside each diquark, which ensures that the resulting pentaquark parity is positive. The heavy antiquark is a $SU(3)_f$ flavor singlet. Hence the heavy pentaquarks containing an anticharm or antibottom form a $SU(3)$ flavor antisextet as shown in Fig. 1. The flavor wave functions of all members of $\bar{\mathbf{6}}_\mathbf{f}$ are listed in Table I.

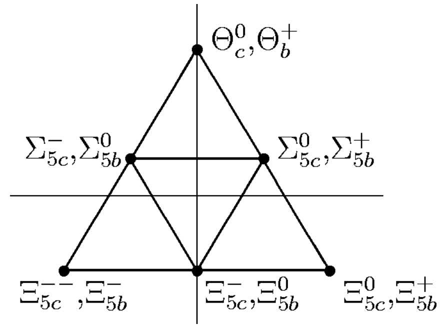

FIG. 1. The six members of the $SU(3)$ flavor antisextet.

The magnetic moment of a compound system is the sum of orbital and spin contributions from all of its constituents. Only the orbital motion of the scalar diquarks contributes to the pentaquark magnetic moment. Each diquark's orbital angular momentum reads

$$\mathbf{l}_1=\mathbf{r}_1\times\mathbf{p}_1=\left(\mathbf{R}+\frac{m_2}{m_1+m_2}\mathbf{r}\right)\times\left(\frac{m_1}{m_1+m_2}\mathbf{P}+\mathbf{p}\right), \quad (1)$$

where $\mathbf{R}\equiv(m_1\mathbf{r}_1+m_2\mathbf{r}_2)/(m_1+m_2)$ is the position of the center of mass of the diquark-diquark system and $\mathbf{r}\equiv\mathbf{r}_1-\mathbf{r}_2$ is the relative position between the two diquarks. Since the pentaquark is a bound state of multi-quarks, $\langle\mathbf{p_i}\rangle=0$. So we have $\langle\mathbf{P}\rangle=\langle\mathbf{p_1}\rangle+\langle\mathbf{p_2}\rangle=0$ and $\langle\mathbf{p}\rangle=\langle(m_2\mathbf{p}_1-m_1\mathbf{p}_2)/(m_1+m_2)\rangle=0$, from which we get

TABLE I. Flavor wave functions in Jaffe and Wilczek's model [20]. $[q_1q_2][q_3q_4]_+=\sqrt{\frac{1}{2}}([q_1q_2][q_3q_4]+[q_3q_4][q_1q_2])$ or $[q_1q_2]^2=[q_1q_2][q_1q_2]$ is the diquark-diquark part.

Pentaquarks	(Y,I,I_3)	Flavor wave functions ($\overline{Q}=\overline{c}$ or $\overline{b}$)
Θ_c^0,Θ_b^+	$(\frac{4}{3},0,0)$	$[ud]^2\overline{Q}$
$\Sigma_{5c}^0,\Sigma_{5b}^+$	$(\frac{1}{3},\frac{1}{2},\frac{1}{2})$	$[ud][us]_+\overline{Q}$
$\Sigma_{5c}^-,\Sigma_{5b}^0$	$(\frac{1}{3},\frac{1}{2},-\frac{1}{2})$	$[ud][ds]_+\overline{Q}$
Ξ_{5c}^0,Ξ_{5b}^+	$(-\frac{2}{3},1,1)$	$[us]^2\overline{Q}$
Ξ_{5c}^-,Ξ_{5b}^0	$(-\frac{2}{3},1,)$	$[us][ds]_+\overline{Q}$
Ξ_{5c}^{--},Ξ_{5b}^-	$(-\frac{2}{3},1,-1)$	$[ds]^2\overline{Q}$

$$\langle \mathbf{l}_1\rangle = \frac{m_2}{m_1+m_2}\langle \mathbf{r}\times\mathbf{p}\rangle + \frac{m_1}{m_1+m_2}\langle \mathbf{R}\times\mathbf{P}\rangle$$

$$= \frac{m_2}{m_1+m_2}\langle \mathbf{l}\rangle + \frac{m_1}{m_1+m_2}\langle \mathbf{L}\rangle, \qquad (2)$$

where $\mathbf{l}=\mathbf{r}\times\mathbf{p}$ is the diquark-diquark's relative orbital angular momentum. $\mathbf{L}=\mathbf{R}\times\mathbf{P}$ is the diquark-diquark's orbital angular momentum for the motion of center of mass. In the center of mass frame $\langle \mathbf{L}\rangle=0$. Therefore we get

$$\langle \mathbf{l}_1\rangle = \frac{m_2}{m_1+m_2}\langle \mathbf{l}\rangle, \qquad (3)$$

$$\langle \mathbf{l}_2\rangle = \frac{m_1}{m_1+m_2}\langle \mathbf{l}\rangle. \qquad (4)$$

For the magnetic moment of the diquark-diquark system we have

$$\mu_l \vec{l} = \frac{m_2\mu_1}{m_1+m_2}\vec{l} + \frac{m_1\mu_2}{m_1+m_2}\vec{l},$$

$$\mu_l = \frac{m_2\mu_1}{m_1+m_2} + \frac{m_1\mu_2}{m_1+m_2}, \qquad (5)$$

where $\mu_i \equiv e_i/2m_i$ is the magneton of the i-th diquark. $\vec{l}=\vec{1}$ is the angular moment between diquarks.

Now we can write down the magnetic moment of a $J^P = \frac{1}{2}^+$ pentaquark as follows:

$$\mu = \left\langle 2\mu_{\bar{Q}}\frac{\vec{1}}{2} + \mu_l \vec{l}\right\rangle \left(J_z = \frac{1}{2}\right)$$

$$= \left(\left\langle 1 0 \frac{1}{2}\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2 - \left\langle 1 1 \frac{1}{2} -\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2\right)$$

$$\times \mu_{\bar{Q}} + \left\langle 1 1 \frac{1}{2} -\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2 \mu_l$$

$$= -\frac{1}{3}\mu_{\bar{Q}} + \frac{2}{3}\mu_l. \qquad (6)$$

Similarly, for $J^P=\frac{3}{2}^+$ pentaquarks, we have

$$\mu = \left\langle 2\mu_{\bar{Q}}\frac{\vec{1}}{2} + \mu_l \vec{l}\right\rangle \left(J_z = \frac{3}{2}\right)$$

$$= \left\langle 1 1 \frac{1}{2}\frac{1}{2}\middle|\frac{3}{2}\frac{3}{2}\right\rangle^2 \mu_{\bar{Q}} + \left\langle 1 1 \frac{1}{2}\frac{1}{2}\middle|\frac{3}{2}\frac{3}{2}\right\rangle^2 \mu_l$$

$$= \mu_{\bar{Q}} + \mu_l. \qquad (7)$$

We use heavy quark masses $m_c = 1710$ MeV, $m_b = 5050$ MeV from Ref. [22] and diquark masses $m_{ud} = 420$ MeV, $m_{us}=m_{ds}=600$ MeV from Ref. [20] to compute the magnetic moments of these $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$ pentaquarks. The numerical results are summarized in Table II.

TABLE II. Numerical results of the $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$ heavy pentaquark magnetic moments in Jaffe and Wilczek's model with $m_{ud}=420$ MeV, $m_{us}=m_{ds}=600$ MeV from Ref. [20].

	Q=c		Q=b	
(Y,I,I_3)	$J^P=\frac{1}{2}^+$	$J^P=\frac{3}{2}^+$	$J^P=\frac{1}{2}^+$	$J^P=\frac{3}{2}^+$
$(\frac{4}{3},0,0)$	0.62	0.38	0.48	0.81
$(\frac{1}{3},\frac{1}{2},\frac{1}{2})$	0.56	0.29	0.41	0.71
$(\frac{1}{3},\frac{1}{2},-\frac{1}{2})$	0.13	−0.36	−0.015	0.071
$(-\frac{2}{3},1,1)$	0.47	0.16	0.33	0.58
$(-\frac{2}{3},1,0)$	−0.052	−0.63	−0.19	−0.20
$(-\frac{2}{3},1,-1)$	−0.57	−1.41	−0.72	−0.98

Jaffe and Wilczek estimated Θ_c^0 and Θ_b^+ masses by replacing the $\bar{s}$ in the Θ^+ with $\bar{c}$ and $\bar{b}$ respectively. The cost of this replacement is roughly the mass difference between Λ_c and Λ baryons. The $[ud]$ diquark in the Λ_c and Λ experiences nearly the same environment as in Θ_c^0 and Θ^+ pentaquarks, especially when the $[ud]$ diquark is viewed as a tightly bound entity. Thus they got

$$M(\Theta_c^0) = M(\Theta^+) + [M(\Lambda_c) - M(\Lambda)]$$

$$M(\Theta_b^+) = M(\Theta^+) + [M(\Lambda_b^+) - M(\Lambda)].$$

Here we extend their formalism to estimate the masses of all the other members of $\bar{\mathbf{6}}_\mathbf{f}$. The heavy pentaquarks in the same isospin multiplet are degenerate:

$$M(\Sigma_{5c}^0) = M(N_s^+) + [M(\Lambda_c) - M(\Lambda)],$$

$$M(\Sigma_{5b}^+) = M(N_s^+) + [M(\Lambda_b^+) - M(\Lambda)].$$

Here N_s^+ has a quark content $|[ud][us]_+\bar{s}\rangle$. Its mass was estimated to be around 1700 MeV [20]. For the heavy pentaquark containing two strange quarks, we use

$$M(\Xi_{5c}^0) = M(\Xi_5^{--}) + [M(\Xi_c) - M(\Sigma)],$$

$$M(\Xi_{5b}^+) = M(\Xi_5^{--}) + [M(\Xi_b) - M(\Sigma)].$$

We use 1860 MeV for the Ξ_5^{--} mass from NA49 experiment [9].

Another simple way to estimate Ξ_{5c}^0 (Ξ_{5b}^+) mass is

$$M(\Xi_{5c}^0) = M(\Theta^+) + 2[m_{us} - m_{ud}] + m_c - m_s,$$

$$M(\Xi_{5b}^+) = M(\Theta^+) + 2[m_{us} - m_{ud}] + m_b - m_s.$$

With $m_{ud}=420$ MeV, $m_{us}=600$ MeV [20], $m_c = 1710$ MeV and $m_b=5050$ MeV [22], the Ξ_{5c} and Ξ_{5b}^+ mass from the second estimate is 3110 and 5460 MeV respectively, which is only 25 MeV below those from the first estimate. Given that the errors of both estimates are around

TABLE III. Masses of all members of $\overline{\mathbf{6}}_{\mathbf{f}}$ in Jaffe and Wilczek's model.

$\overline{Q}=\overline{c}$	Masses	$\overline{Q}=\overline{b}$	Mass
Θ_c^0	2710 MeV	Θ_b^+	6050 MeV
$\Sigma_{5c}^-,\Sigma_{5c}^0$	2870 MeV	$\Sigma_{5b}^0,\Sigma_{5b}^+$	6210 MeV
$\Xi_{5c}^{--},\Xi_{5c}^-,\Xi_{5c}^0$	3135 MeV	$\Xi_{5b}^-,\Xi_{5b}^0,\Xi_{5b}^+$	6475 MeV

100 MeV, these two approaches yield quite consistent results, which are collected in Table III.

The mass splitting between $J^P=\frac{1}{2}^+$ light pentaquarks and their $J^P=\frac{3}{2}^+$ partners are estimated to around tens of MeVs according to Dudek and Close [23]. The mass splitting between $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$ multiplets scales as the product of the inverse constituent mass. Hence for heavy pentaquarks, the mass splitting between $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$ multiplets will be even smaller, around 10 MeV.

III. HEAVY PENTAQUARK STATES IN KARLINER AND LIPKIN'S MODEL

In Karliner and Lipkin's (KL) model [22], the pentaquark is divided into two color non-singlet clusters: a scalar diquark and a triquark. There is a *P*-wave excitation between the two clusters. The angular momentum barrier between the two clusters prevents them from rearranging into the usual baryon and meson system.

The two quarks in the triquark are in the symmetric $\mathbf{6}_{\mathbf{c}}$ representation. They couple with the antiquark to form an $SU(3)_c$ triplet $\mathbf{3}_c$. The two quarks are in the antisymmetric flavor $\overline{\mathbf{3}}_{\mathbf{f}}$ representation. The anticharm or antibottom is a $SU(3)_f$ flavor singlet. Hence the triquark belongs to a flavor antitriplet. The spin wave function of the two quarks inside the triquark is symmetric. The spin of the triquark is one half.

The direct product of the $\overline{\mathbf{3}}_{\mathbf{f}}$ of diquark and the $\overline{\mathbf{3}}_{\mathbf{f}}$ of triquark leads to $\overline{\mathbf{6}}_{\mathbf{f}}$ and $\mathbf{3}_{\mathbf{f}}$ pentaquarks. There is one orbital angular momentum $L=1$ between the diquark and the triquark. The resulting J^P of the pentaquark can be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. We list the flavor wave functions of these two multiplets in Tables IV and V.

TABLE IV. Flavor wave functions of the antisextet pentaquarks in Karliner and Lipkin's model [22]. Y, I and I_3 are hypercharge, isospin and the third component of isospin respectively. $\{q_1q_2\overline{Q}\}\equiv[q_1q_2]\overline{Q}$ is the triquark's flavor wave function.

pentaquarks	(Y,I,I_3)	Flavor wave functions
Θ_c^0,Θ_b^+	$(\frac{4}{3},0,0)$	$[ud]\{ud\overline{Q}\}$
$\Sigma_{5c}^0,\Sigma_{5b}^+$	$(\frac{1}{3},\frac{1}{2},\frac{1}{2})$	$\sqrt{\frac{1}{2}}([ud]\{us\overline{Q}\}+[us]\{ud\overline{Q}\})$
$\Sigma_{5c}^-,\Sigma_{5b}^0$	$(\frac{1}{3},\frac{1}{2},-\frac{1}{2})$	$\sqrt{\frac{1}{2}}([ud]\{ds\overline{Q}\}+[ds]\{ud\overline{Q}\})$
Ξ_{5c}^0,Ξ_{5b}^+	$(-\frac{2}{3},1,1)$	$[us]\{us\overline{Q}\}$
Ξ_{5c}^-,Ξ_{5b}^0	$(-\frac{2}{3},1,0)$	$\sqrt{\frac{1}{2}}([us]\{ds\overline{Q}\}+[ds]\{us\overline{Q}\})$
Ξ_{5c}^{--},Ξ_{5b}^-	$(-\frac{2}{3},1,-1)$	$[ds]\{ds\overline{Q}\}$

TABLE V. Flavor wave functions of the triplet heavy pentaquarks in Karliner and Lipkin's model [22]. Notations are the same as in Table IV.

Pentaquarks	(Y,I,I_3)	Flavor wave functions
$\Sigma_{5c}'^0,\Sigma_{5b}'^+$	$(\frac{1}{3},\frac{1}{2},\frac{1}{2})$	$\sqrt{\frac{1}{2}}([ud]\{us\overline{Q}\}-[us]\{ud\overline{Q}\})$
$\Sigma_{5c}'^-,\Sigma_{5b}'^0$	$(\frac{1}{3},\frac{1}{2},-\frac{1}{2})$	$\sqrt{\frac{1}{2}}([ud]\{ds\overline{Q}\}-[ds]\{ud\overline{Q}\})$
$\Xi_{5c}'^-,\Xi_{5b}'^0$	$(-\frac{2}{3},0,0)$	$\sqrt{\frac{1}{2}}([us]\{ds\overline{Q}\}-[ds]\{us\overline{Q}\}$

The intrinsic magnetic moment of the triquark is defined as

$$g_{tri}\mu_{tri}\frac{\vec{1}}{2}=2\mu_{q_1}\frac{\vec{1}}{2}+2\mu_{q_2}\frac{\vec{1}}{2}+2\mu_{\overline{Q}}\frac{\vec{1}}{2}. \tag{8}$$

From the spin structure of the triquark, we get

$$\frac{1}{2}g_{tri}\mu_{tri}=\left(\left\langle 10\frac{1}{2}\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2-\left\langle 11\frac{1}{2}-\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2\right)\mu_{\overline{Q}} +\left\langle 11\frac{1}{2}-\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2(\mu_{q_1}+\mu_{q_2}). \tag{9}$$

For the orbital part we have

$$\mu_l=\frac{m_{tri}\mu_{di}+m_{di}\mu_{tri}}{m_{tri}+m_{di}}, \tag{10}$$

where m_{di} is the mass of the diquark, m_{tri} is the mass of the triquark.

For $J^P=\frac{1}{2}^+$, the magnetic moment of the pentaquark is

$$\mu=\left(\left\langle 10\frac{1}{2}\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2-\left\langle 11\frac{1}{2}-\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2\right)\frac{1}{2}g_{tri}\mu_{tri} +\left\langle 11\frac{1}{2}-\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right\rangle^2\mu_l =\frac{2}{3}\mu_l-\frac{1}{3}\times\left(\frac{1}{2}g_{tri}\mu_{tri}\right). \tag{11}$$

For $J^P=\frac{3}{2}^+$, we have

$$\mu=\left\langle 11\frac{1}{2}\frac{1}{2}\middle|\frac{3}{2}\frac{3}{2}\right\rangle^2\mu_l+\left\langle 11\frac{1}{2}\frac{1}{2}\middle|\frac{3}{2}\frac{3}{2}\right\rangle^2\frac{1}{2}g_{tri}\mu_{tri} =\mu_l+\frac{1}{2}g_{tri}\mu_{tri}. \tag{12}$$

The results are listed in Table VI. We use $m_u=m_d=360$ MeV, $m_s=500$ MeV while $m_c=1710$ MeV, $m_b=5050$ MeV as in Ref. [22]. The mass of each diquark (triquark) is the sum of its constituent mass.

Karliner and Lipkin have estimated the masses of Θ^+ and its heavy flavor analogs [22]. The mass of a pentaquark comes mainly from the masses of its constituent quarks, the color-spin hyperfine interaction and the P-wave excitation

TABLE VI. Numerical results of heavy pentaquark magnetic moments in unit of μ_N in Karliner and Lipkin's model.

Pentaquarks	Q=c		Q=b	
	$J^P=\frac{1}{2}^+$	$J^P=\frac{3}{2}^+$	$J^P=\frac{1}{2}^+$	$J^P=\frac{3}{2}^+$
Θ_c^0,Θ_b^+	−0.031	1.01	0.079	0.96
$\Sigma_{5c}^0,\Sigma_{5c}^{\prime 0},\Sigma_{5b}^+,\Sigma_{5b}^{\prime +}$	−0.079	1.06	0.030	1.00
$\Sigma_{5c}^-,\Sigma_{5c}^{\prime -},\Sigma_{5b}^0,\Sigma_{5b}^{\prime 0}$	−0.084	−0.26	−0.0028	−0.35
Ξ_{5c}^0,Ξ_{5b}^+	−0.13	1.11	−0.020	1.05
$\Xi_{5c}^-,\Xi_{5c}^{\prime -},\Xi_{5b}^0,\Xi_{5b}^{\prime 0}$	−0.14	−0.22	−0.054	−0.30
Ξ_{5c}^{--},Ξ_{5b}^-	−0.15	−1.54	−0.089	−1.66

between the two clusters. The first two parts can be estimated by comparing them with a relevant baryon-meson system that have the same quark contents as this pentaquark. The hyperfine energy difference between these two systems can be figured out using the $SU(6)$ color-spin algebra and has been given as $-[(1+\zeta_Q)/12][M(\Delta)-M(N)]$ by Karliner and Lipkin [22], where $\zeta_Q \equiv m_u/m_Q$. The P-wave excitation energy δE^P is approximate to the mass difference between $D_s(2319)$ and $D_s^*(2112)$. This approximation is based on the observation that the reduced mass of the diquark-triquark system is close to that of the D_s [22].

Following this formalism we estimate the masses of other members of $\overline{\mathbf{6}}_{\mathbf{f}}$ and $\mathbf{3}_{\mathbf{f}}$. We have

$$
\begin{aligned}
M(\Sigma_{5c}^-)=M(\Sigma_{5c}^0)=M(\Sigma_{5c}^{\prime -})=M(\Sigma_{5c}^{\prime 0}) \\
&=\frac{1}{2}[M(N)+M(D_s)+M(\Sigma)+M(D)] \\
&-\frac{1+\zeta_c}{12}[M(\Delta)-M(N)]+\delta E^P,
\end{aligned}
$$

$$
\begin{aligned}
M(\Sigma_{5b}^0)=M(\Sigma_{5b}^+)=M(\Sigma_{5b}^{\prime 0}) \\
&=M(\Sigma_{5b}^{\prime +})=\frac{1}{2}[M(N)+M(B_s)+M(\Sigma) \\
&+M(B)]-\frac{1+\zeta_b}{12}[M(\Delta)-M(N)]+\delta E^P
\end{aligned}
$$

and

$$
\begin{aligned}
M(\Xi_{5c}^{--})=M(\Xi_{5c}^-)=M(\Xi_{5c}^{\prime -})=M(\Xi_{5c}^0)=M(\Sigma) \\
&+M(D_s)]-\frac{1+\zeta_c}{12}[M(\Delta)-M(N)]+\delta E^P,
\end{aligned}
$$

$$
\begin{aligned}
M(\Xi_{5b}^-)=M(\Xi_{5b}^0)=M(\Xi_{5b}^{\prime 0})=M(\Xi_{5b}^+) \\
&=M(\Sigma)+M(B_s)]-\frac{1+\zeta_b}{12}[M(\Delta)-M(N)] \\
&+\delta E^P.
\end{aligned}
$$

The numerical values of the masses of all members of $\overline{\mathbf{6}}_{\mathbf{f}}$ and $\mathbf{3}_{\mathbf{f}}$ are presented in Table VII.

IV. DISCUSSIONS

In this paper we have studied the masses and magnetic moments of heavy pentaquarks in the framework of two clustered quark models. There is only a $SU(3)$ flavor antisextet in Jaffe and Wilczek's model. In contrast, an additional flavor triplet exists in Karliner and Lipkin's diquark and triquark model. The masses and magnetic moments of these triplet pentaquarks are equal to their antisextet partners in KL's model. For the light pentaquarks, all the above two clustered quark models predicted the existence of an antidecuplet and an accompanying octet.

Another dramatic difference lies in the prediction of the magnetic moments of the $J^P=\frac{1}{2}^+$ heavy anti-sextet pentaquarks. The magnetic moments of the $J^P=\frac{1}{2}^+$ heavy antisextet is tiny and much smaller in KL's model than those in JW's model. There is strong cancellation between the magnetic moment of the triquark and the orbital magnetic moment in a $J^P=\frac{1}{2}^+$ anti-sextet when they are combined by Clebsch-Gordan coefficients [cf. Eq. (11)]. This cancellation is almost exact for bottom pentaquarks. For the magnetic moments of the $J^P=\frac{3}{2}^+$ anti-sextet, there is also significant difference between KL's model and JW's models.

The third difference is that the masses of the pentaquarks in KL's model are about 300 MeV larger than those in JW's model. The reason is that Karliner and Lipkin assumed that the diquark (triquark) mass is simply the sum of its constituents. In contrast, the diquark mass is made much lower than the sum of the two quarks through the strong correlation between light quarks when they are in antisymmetric configuration in JW's model [20].

In JW's models, the masses of heavy pentaquarks are about 100 MeV below the threshold of the strong decay modes if one believes our rough estimate. Only isospin violating strong decays, electromagnetic decays and weak decays can occur in this case. These heavy anti-sextet members should be stable. In the case that their masses are underestimated by around 100 MeV in our calculation, they are still barely above threshold. Severe phase space suppression will make them very narrow even if they have strong decays.

TABLE VII. Masses of all members of $\overline{\mathbf{6}}_{\mathbf{f}}$ and $\mathbf{3}_{\mathbf{f}}$ in Karliner and Lipkin's model.

$\overline{Q}=\overline{c}$	Mass	$\overline{Q}=\overline{b}$	Mass
Θ_c^0	2990 MeV	Θ_b^+	6400 MeV
$\Sigma_{5c}^-(\Sigma_{5c}^{\prime -})$, $\Sigma_{5c}^0(\Sigma_{5c}^{\prime 0})$	3165 MeV	$\Sigma_{5b}^0(\Sigma_{5b}^{\prime 0})$, $\Sigma_{5b}^+(\Sigma_{5b}^{\prime +})$	6570 MeV
$\Xi_{5c}^{--},\Xi_{5c}^-(\Xi_{5c}^{\prime -}),\Xi_{5c}^0$	3340 MeV	$\Xi_{5b}^-,\Xi_{5b}^0(\Xi_{5b}^{\prime 0})$, Ξ_{5b}^+	6740 MeV

On the other hand, the pentaquarks in KL's model lie above threshold and have strong decay modes. For example, the strong decay $\Xi^{+}_{5b} \rightarrow B^{0}_{s}\Sigma^{+}$ and $\Xi^{+}_{5b} \rightarrow B^{+}\Xi^{0}$ may occur in KL's model. However, they may still be narrow states as the Θ^{+} pentaquark even if strong decay modes exist.

The presence of additional triplet states, different magnetic moments, different masses and decay modes will play a role in distinguishing these two clustered quark models. Future experiments at CLEOc, BESIII, BELLE and LEP will be able to find these very interesting states if they really exist. Hopefully the present study will somehow contribute to the experimental search.

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